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Analyses of k_t distributions at RHIC by means of some selected statistical and stochastic models

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Abstract. The new data on k_t distributions obtained at RHIC are analyzed by means of selected models of statistical and stochastic origin in order to estimate their importance in providing new information on the hadronization process, in particular on the value of the temperature at freeze-out to the hadronic phase.

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1 Introduction

Very recently high k_t distributions at RHIC have been reported in [1–3]. These data are of potentially high interest as a possible source of information on the conditions existing at the freeze-out in heavy-ion collisions. This resulted in a number of works, mostly of statistical or thermal origin [4], stressing different possible dynamical aspects, like the role of resonances or the flow phenomenon. In our work we would like to show that one can account summarily for such (and others) effects considered in the literature by using simple minimal extensions of the known statistical or stochastic models, which were already successfully applied in other analysis of experimental data. They are as follows.
(i) The modified statistical model inspired by Tsallis statistics [5], which generalizes the usual Boltzmann–Gibbs statistics to non-extensive systems parametrized by a non-extensivity parameter q (for $q \to 1$ one returns to the usual Boltzmann–Gibbs extensive scenario); it has been already successfully used in this context [6–8]. The parameter q summarizes in such an approach all deviations from the Boltzmann–Gibbs statistics including those caused by flow phenomena and resonances [4].
(ii) A suitable adaptation of the recently proposed model derived from the Fokker–Planck equation for the Orstein–Uhlenbeck (O-U) process [9–11] but this time used in the transverse rapidity space, i.e., for $y_t = \frac{1}{2}\ln[(m_t+k_t)/(m_t-k_t)]$ (where $m_t = \sqrt{m^2+\langle k_t\rangle^2}$), in which one allows the mass m to be treated as free parameter in order to account for some specific features of the data (like the flow phenomenon) which cannot be explained in a usual way.

As a kind of historical reference point we shall use the classical statistical model developed a long time ago by Hagedorn [12] in which the transverse momentum distribution of the produced secondaries is given by the following formula [13] (with T_0 being the parameter identified with temperature, T_h denoting the so-called Hagedorn temperature [12,13] and m_π being the pion mass):

$$\frac{d^2\sigma}{2\pi k_t dk_t} = C\int_{m_\pi}^{\infty} dm\rho(m)\sqrt{m^2+k_t^2}K_1\left(\frac{\sqrt{m^2+k_t^2}}{T_0}\right); \tag{1}$$

$$\rho(m) = \frac{e^{m/T_h}}{(m^2+m_0^2)^{5/4}}. \tag{2}$$

As one can see in Fig. 1 although fits to the k_t distributions at $\sqrt{s_{NN}} = 200$ GeV obtained by the BRAHMS Collaboration [1] are quite good, they start to deviate from the data at the highest values of k_t and became very bad there, which is very clearly seen in Fig. 2 where we show our fits to the STAR data [2] covering a larger span of transverse momenta. Although one can argue that for such large values of k_t the statistical approach must give way to some more detailed dynamical calculations [4], there are examples that suitable modifications of the statistical approach can lead to quite reasonable results in leptonic, hadronic and nuclear collisions. What we have in mind here are some non-extensive generalizations of the statistical model as discussed in [6–8] and some specific realization of the stochastic approach as proposed by [9,10,14]. In what follows we shall therefore apply these two methods to nuclear data of [1–3].

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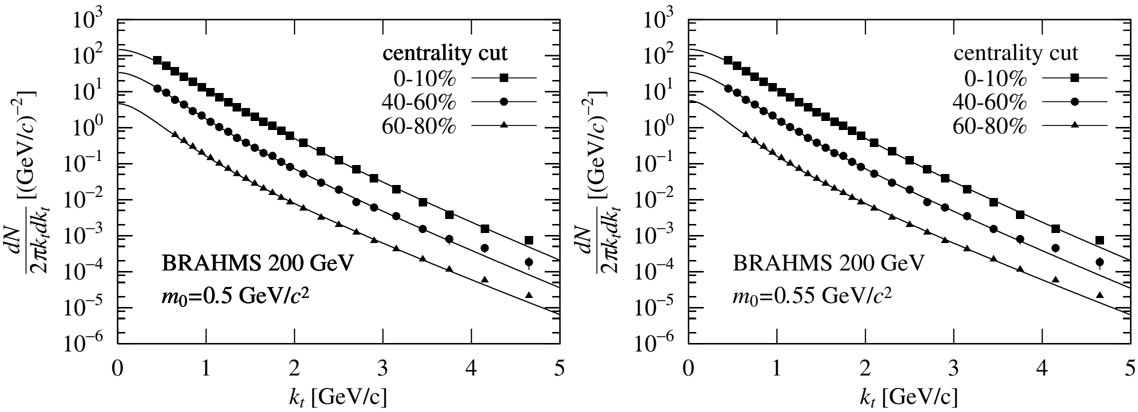

Fig. 1. Results of the application of the simple statistical model, cf. (1), to the data for k_t distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by the BRAHMS Collaboration [1]

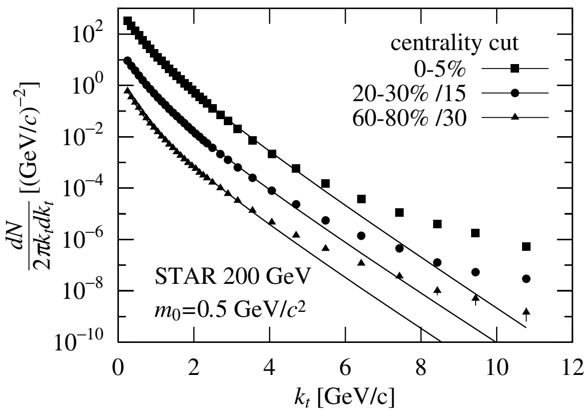

Fig. 2. Results of the application of the simple statistical model, cf. (1), to the data for k_t distributions at $\sqrt{s_{NN}} = 200$ GeV measured for different centralities by the STAR Collaboration [2]

In next section we shall analyze the data using the non-extensive generalization of the statistical model by means of Tsallis statistics. In Sect. 3 we shall analyze the data using the stochastic approach in transverse rapidity space. Our conclusions are presented in Sect. 4.

Table 1. Values of parameters C, T_h and T_0 in (1) used to obtain the results presented in Figs. 1 and 2. The values of χ^2/n.d.f. for the BRAHMS data are the same for all centralities and equal to 19.3/23 and 18.3/23 for $m_0 = 0.5$ and 0.55 GeV, respectively. For the STAR data they are equal 532/32 for C.C. = 0–5%, 249/32 for C.C. = 20–30% and 308/32 for C.C. = 60–80%

BRAHMS Coll. [1] $m_0 = 0.5$ GeV (fixed)			
C.C. (%)	C	T_h (GeV)	T_0 (GeV)
0–10	177 ± 11	0.180 ± 0.007	0.169 ± 0.006
10–20	127 ± 9	0.172 ± 0.008	0.162 ± 0.006
20–40	83 ± 7	0.156 ± 0.008	0.149 ± 0.007
40–60	44 ± 5	0.133 ± 0.010	0.128 ± 0.009
60–80	177 ± 11	0.095 ± 0.0001	0.093 ± 0.0001
BRAHMS Coll. [1] $m_0 = 0.55$ GeV (fixed)			
0–10	204 ± 13	0.172 ± 0.008	0.162 ± 0.006
10–20	146 ± 10	0.163 ± 0.008	0.155 ± 0.006
20–40	96 ± 8	0.148 ± 0.008	0.142 ± 0.007
40–60	51 ± 6	0.124 ± 0.010	0.121 ± 0.009
60–80	204 ± 13	0.075 ± 0.00007	0.075 ± 0.0001
STAR Coll. [2] $m_0 = 0.5$ GeV (fixed)			
0–5	816 ± 15	0.086 ± 0.0001	0.085 ± 0.0001
20–30	382 ± 7	0.077 ± 0.0001	0.076 ± 0.0001
60–80	106 ± 2	0.037 ± 0.00001	0.037 ± 0.00001

2 Analysis of k_t distributions by generalized statistical model based on Tsallis statistics

In many fields in physics which use statistical and stochastic approaches as their tools, it was recognized since some time ago already that the usual Boltzmann–Gibbs approach encounters serious problems when applied to systems possessing memory effects, correlations (especially long-range correlations but also those caused by the production of resonances in multiparticle production processes or by the flow effects present there) or which phase space has some (multi) fractal structure [5]. Such systems are all, in a sense, *small*, by which we mean that the effective range of correlations they experience is of the order of the dimension of the system itself. Therefore they will not show the property of extensivity leading to the Boltzmann–Gibbs form of entropy, which is the basis of any statistical or stochastical model. One can therefore argue that in such cases one has to resort to some dynamical approach in which the effects mentioned above would be properly accounted for. The problem is, however, that there is no unique model of this type and usually several approaches are competing among themselves in describing the experimental data. The other possibility is to realize that most probably our system is not extensive (in the abovementioned sense) and that this fact should be accounted for by using a non-extensive form of entropy, for example the so-called Tsallis entropy [5]. It turns out that such situations are encountered also in the domain of multiparticle production processes at high energy collisions (cf. [6], to which we refer for all details). In fact, there already exist a number of detailed analyses using a non-extensive approach ranging from k_t distributions in e^+e^- annihilations [7] and in $p+\bar{p}$ collisions [8] to rapidity distributions in some selected reactions [6]. In [7,8] a kind of non-extensive q-version of the Hagedorn approach has been used whereas [6] were exploring the information theoretical approach to statistical models including as an option also its non-extensive version[1].

In our work we shall apply the Tsallis formalism, treated as the simplest possible extension of the usual statistical approach with parameter q (the so-called non-extensivity parameter or entropic index) summarizing deviations from the usual statistical approach (without, however, specifying their dynamical origin). It leads to (T_0 denotes the temperature)

$$\frac{\mathrm{d}^2\sigma}{2\pi k_t \mathrm{d}k_t} = C\int_0^\infty \left[1-(1-q)\frac{\sqrt{k_t^2+k_l^2+m^2}}{T_0}\right]^Q \mathrm{d}k_l. \tag{3}$$

There exist two different formulations leading to slightly different forms of the parameter Q.

(a) In the first one uses the so-called escort probability distributions [19], $P_i = p_i^q/\sum_i p_i^q$ (cf., for example, the analysis of k_t distributions in e^+e^- annihilations [7] or in $p\bar{p}$ collisions [8]); in this case $Q = q/(1-q)$.

(b) In the second approach one uses the normal definition of the probabilities resulting in $Q = 1/(1-q)$. In this case, as shown in [15,17], the parameter q is given by the normalized variance of all intrinsic fluctuations present in the hadronizing system under consideration:

$$q = 1+\omega = 1+\left(\langle\beta^2\rangle - \langle\beta\rangle^2\right)/\langle\beta\rangle^2. \tag{4}$$

This conjecture originates from the observation that

$$[1-(1-q)\beta_0 H_0]^{\frac{1}{1-q}} = \int_0^\infty \mathrm{e}^{-\beta H_0} f(\beta)\mathrm{d}\beta, \tag{5}$$

where $f(\beta)$ describes fluctuation of parameter β and has the form of the Gamma distribution [15, 17] (in our case $H_0 = \sqrt{k_l^2+k_t^2+m^2}$ and fluctuations are in temperature, i.e., $\beta = 1/T$ and $\beta_0 = \langle\beta\rangle$ with respect to $f(\beta)$)[2].

We have analyzed the BRAHMS [1], STAR [2] and PHENIX [3] data and our results are shown in Fig. 3 and in Table 2. It turns out that both forms of the parameter Q in (3) result in practically identical curves; therefore here we are showing only the results obtained for $Q = q/(1-q)$. The values of the parameters are also very close to each other with a tendency of C, T_0 and q estimated by using $Q = 1/(1-q)$ being slightly bigger than those obtained for $Q = q/(1-q)$. It is worth to stress at this point that such a comparison of these two approaches has been made for the first time here and, as one can see from the presented results, it confirms the previous expectation (made in [6]) that in case of only limited phenomenological applications, as is the case of our work, the results from using (3) with $Q = q/(1-q)$ (i.e., parameters $C^{(a)} = c$, $T_0^{(a)} = l$ and $q^{(a)} = q$) are simply connected to those using $Q = 1/(1-q)$ (i.e., to the parameters $C^{(b)} = C$, $T_0^{(b)} = L$ and $q^{(b)} = \hat{Q}$), namely

$$\hat{Q} \simeq 1-\frac{1-q}{q}, \qquad L \simeq \frac{l}{q}, \qquad C \simeq cq. \tag{6}$$

As one can see from Table 2 these relations are indeed satisfied (some small differences present could be attributed to the fact that both sets of results represent results of separate and independent fitting procedures, without making use of (6)). This means therefore that in all phenomenological applications one can use either of the two forms of the parameter Q in (3), and, if necessary, to use (6) to translate results from one scheme to another. In both cases the pion mass value has been used, $m = 0.14\,\mathrm{GeV}$ (and we have checked that additional changes in mass m of the type introduced recently in [14], would not affect the final results as long as m is limited to, say, $m < 0.2\,\mathrm{GeV}$). The estimated fluctuations of the temperature are of the order of 30–45 MeV. It is interesting to observe that these fluctuations are weaker at small centralities and grow for more peripheral collisions matching very nicely similar the fluctuations seen in the $p+p$ data [2] shown here for comparison. One should add here also the result from a similar analysis of the e^+e^- data [7] reporting even higher values of non-extensivity parameter q (reaching the value of $q \simeq 1.2$), i.e., much stronger fluctuations. These results confirm therefore, for the first time, another expectation made in [6] saying that precisely such a trend should be observed. This is because (4) can also be interpreted as being a measure of the *total heat capacity* C_h of the hadronizing system (cf. [6]):

$$\frac{1}{C_h} = \frac{\sigma^2(\beta)}{\langle\beta\rangle^2} = \omega = q-1. \tag{7}$$

As the heat capacity C_h is proportional to the volume, $C_h \sim V$, in our case V would be the volume of the interaction (or hadronization), it is expected to grow with volume and, respectively, q is expected to decrease with V, which is indeed the case if one puts together the results for e^+e^-, $p\bar{p}$ and AA collisions.

3 Analysis of k_t distributions using stochastic approach in y_t space

Whereas the previous approach was concerned with the extension of the purely statistical approach the one presented

[1] It should be mentioned at this point that the proper formulation of the Hagedorn model using Tsallis q-statistics has been proposed in [16]. We shall not pursue this problem here.

[2] It must be mentioned at this point that this suggestion, which in [15] has been derived only for the $q > 1$ case, has been shown to be valid also for the $q < 1$ case [17] and has been extended to the general form of fluctuations leading then to the new concept of *superstatistics* proposed in [18]. The most recent discussion of the physical meaning of the q parameter when applied to multiparticle production processes (and in this context also of the possible origin of statistical formulas as well) with relevant references can be found in [6].

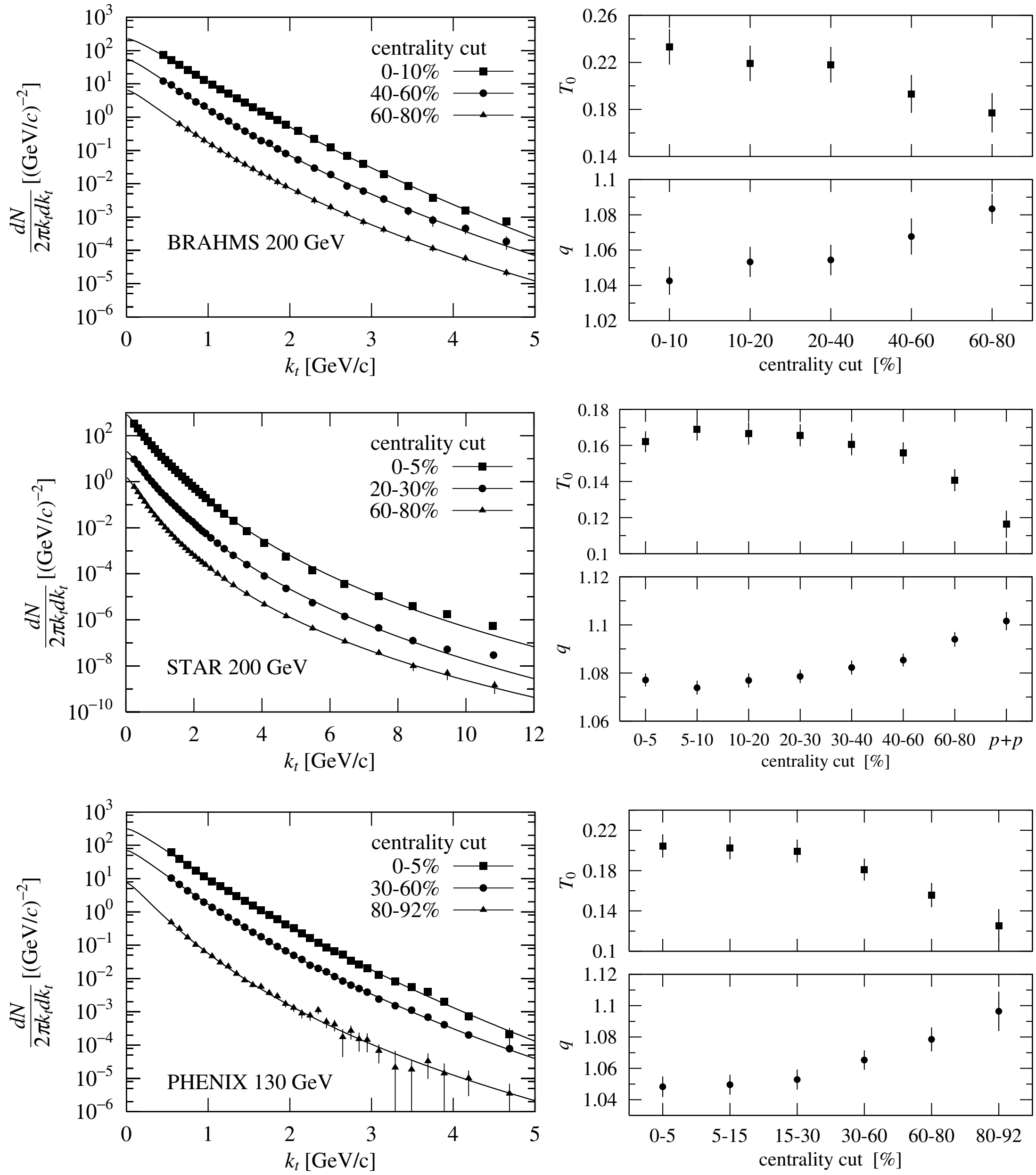

Fig. 3. Results of the application of the non-extensive approach given by (3) with $Q = q/(1-q)$ to the data for k_t distributions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ measured for different centralities by the BRAHMS [1] and STAR [2] Collaborations and to the data at $\sqrt{s_{NN}} = 130\,\mathrm{GeV}$ as measured by the PHENIX Collaboration [3]. The results obtained using $Q = 1/(1-q)$ instead look essentially the same; therefore they are not shown separately. For differences in values of obtained parameters see Table 2

now will go a bit further by modelling the hadronization process by a kind of diffusion mechanism [9,10,14] in which the original energy of the projectiles is being dissipated in some well defined way into a number of produced secondaries occurring in a different part of the phase space[3]. In the case considered here it is the diffusion process taking place in the k_t space. Actually, it turns out that it is more suitable to consider such a diffusion as taking place in the $y_\mathrm{t} = \sinh^{-1}(k_\mathrm{t}/m)$ space. In this case one obtains the following Fokker–Planck equation:

$$\frac{\partial P_\mathrm{t}(y_\mathrm{t}, t)}{\partial t} = \gamma \left[\frac{\partial y_\mathrm{t} P_\mathrm{t}(y_\mathrm{t},\, t)}{\partial y_\mathrm{t}} + \frac{\sigma_\mathrm{t}^2}{2\gamma} \frac{\partial^2 P_\mathrm{t}(y_\mathrm{t}, t)}{\partial y_\mathrm{t}^2} \right]. \quad (8)$$

Its solution can be expressed by a Gaussian distribution,

$$\frac{\mathrm{d}^2\sigma}{2\pi k_\mathrm{t}\mathrm{d}k_\mathrm{t}} = C P_\mathrm{t}(y_\mathrm{t}, t) = \frac{C}{\sqrt{2\pi V_\mathrm{t}^2(t)}} \exp\left[-\frac{y_\mathrm{t}^2}{2V_\mathrm{t}^2(t)} \right], (9)$$

[3] It should be mentioned here that there exist also non-extensive versions of such diffusion process applied to multiparticle production data [20] but we shall not pursue this possibility here.

Table 2. Values of the characteristic parameters used to fit the data on the k_t distributions at different centralities by using the non-extensive approach as given by (3) with $Q = q/(1-q)$ and $Q = 1/(1-q)$ for the data at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ obtained by the BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130\,\mathrm{GeV}$ obtained by the PHENIX Collaboration [3]. For results obtained using $Q = 1/(1-q)$ we provide also explicit values of the corresponding fluctuations of temperature as given by $\Delta T_0 = \sqrt{q-1}\cdot T_0$. The order of magnitude of the corresponding errors for T_0, q and ΔT_0, δT_0, δq and $\delta\Delta T_0$, respectively, are listed below as well. In the analysis we have used errors either as provided by the experiments (for STAR and PHENIX) or assuming a systematic error on the level of 5% (for BRAHMS)

BRAHMS Coll. [1]

	(3) with $Q = q/(1-q)$; $\delta T_0 = 0.005$–0.007, $\delta q = 0.003$–0.005.				(3) with $Q = 1/(1-q)$; $\delta T_0 = 0.005$–0.007, $\delta q = 0.002$–0.004, $\delta\Delta T_0 = 0.001$–0.002.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0–10	11.2/23	1033 ± 78	0.232	1.043	11.2/23	1033 ± 78	0.223	1.041	0.045
10–20	12.9/23	797 ± 66	0.224	1.049	12.9/23	797 ± 65	0.213	1.047	0.046
20–40	12.7/23	525 ± 49	0.215	1.055	12.7/23	525 ± 49	0.204	1.052	0.047
40–60	10.5/23	304 ± 38	0.193	1.067	10.5/23	304 ± 38	0.181	1.063	0.045
60–80	2.85/22	41 ± 5	0.175	1.084	2.85/22	41 ± 5	0.161	1.077	0.045

STAR Coll. [2]

	(3) with $Q = q/(1-q)$; $\delta T_0 = 0.002$–0.003, $\delta q = 0.001$–0.002.				(3) with $Q = 1/(1-q)$; $\delta T_0 = 0.002$–0.003, $\delta q \cong 0.001$, $\delta\Delta T_0 \cong 0.001$.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0–5	170/32	4684 ± 231	0.171	1.071	170/32	4686 ± 231	0.159	1.066	0.041
5–10	68/32	3393 ± 184	0.176	1.068	67.8/32	3393 ± 185	0.165	1.064	0.041
10–20	69/32	2767 ± 144	0.171	1.073	69.2/32	2768 ± 144	0.160	1.068	0.042
20–30	45/32	1928 ± 102	0.169	1.075	44.7/32	1928 ± 102	0.157	1.070	0.042
30–40	44/32	1391 ± 78	0.165	1.078	43.9/32	1391 ± 78	0.153	1.072	0.041
40–60	19/32	896 ± 50	0.153	1.085	19.2/32	896 ± 50	0.141	1.079	0.040
60–80	14/32	414 ± 25	0.137	1.095	14.2/32	413 ± 25	0.125	1.087	0.037
$p+p$	9.7/29	62 ± 7	0.117	1.099	9.62/29	61.9 ± 7.1	0.107	1.090	0.032

PHENIX Coll. [3]

	(3) with $Q = q/(1-q)$; $\delta T_0 = 0.011$–0.016, $\delta q = 0.008$–0.010.				(3) with $Q = 1/(1-q)$; $\delta T_0 = 0.011$–0.016, $\delta q = 0.005$–0.011, $\delta\Delta T_0 = 0.003$–0.005.				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	q	χ^2/n.d.f.	C	T_0 (GeV)	q	ΔT_0 (GeV)
0–5	5.13/29	1694 ± 409	0.201	1.049	5.1/29	1694 ± 411	0.192	1.047	0.042
5–15	3.62/29	1330 ± 316	0.199	1.051	3.6/29	1330 ± 316	0.190	1.048	0.042
15–30	5.55/29	846 ± 206	0.196	1.054	5.6/29	846 ± 206	0.186	1.051	0.042
30–60	2.63/29	433 ± 113	0.178	1.066	2.6/29	433 ± 113	0.167	1.074	0.045
60–80	10.6/29	139 ± 48	0.152	1.080	10.6/29	139 ± 48	0.141	1.062	0.035
80–92	9.10/29	74 ± 45	0.121	1.098	9.1/29	74 ± 42	0.110	1.089	0.033

with[4]

$$2V_t^2(t) = \frac{\sigma_t^2}{\gamma}(1 - \mathrm{e}^{-2\gamma t}). \tag{10}$$

In Fig. 4 we show our results of using (9). It should be noticed that now, following [14], we have regarded the mass m to be a free parameter. Only then we can obtain good agreement with the data. In a sense, the variable mass m corresponds in this approach to the non-extensivity parameter q introduced in Sect. 2 in that it summarily accounts for some additional effects not accounted for by a simple diffusion process (like, for example, the effect of resonances and flow).

In the stochastic approach considered here we do not have direct access to the temperature T_0. It is accessible only if we additionally assume the validity of the Einstein

[4] See also [9]. Actually, (9) is the same as the formula used already a long time ago in [11].

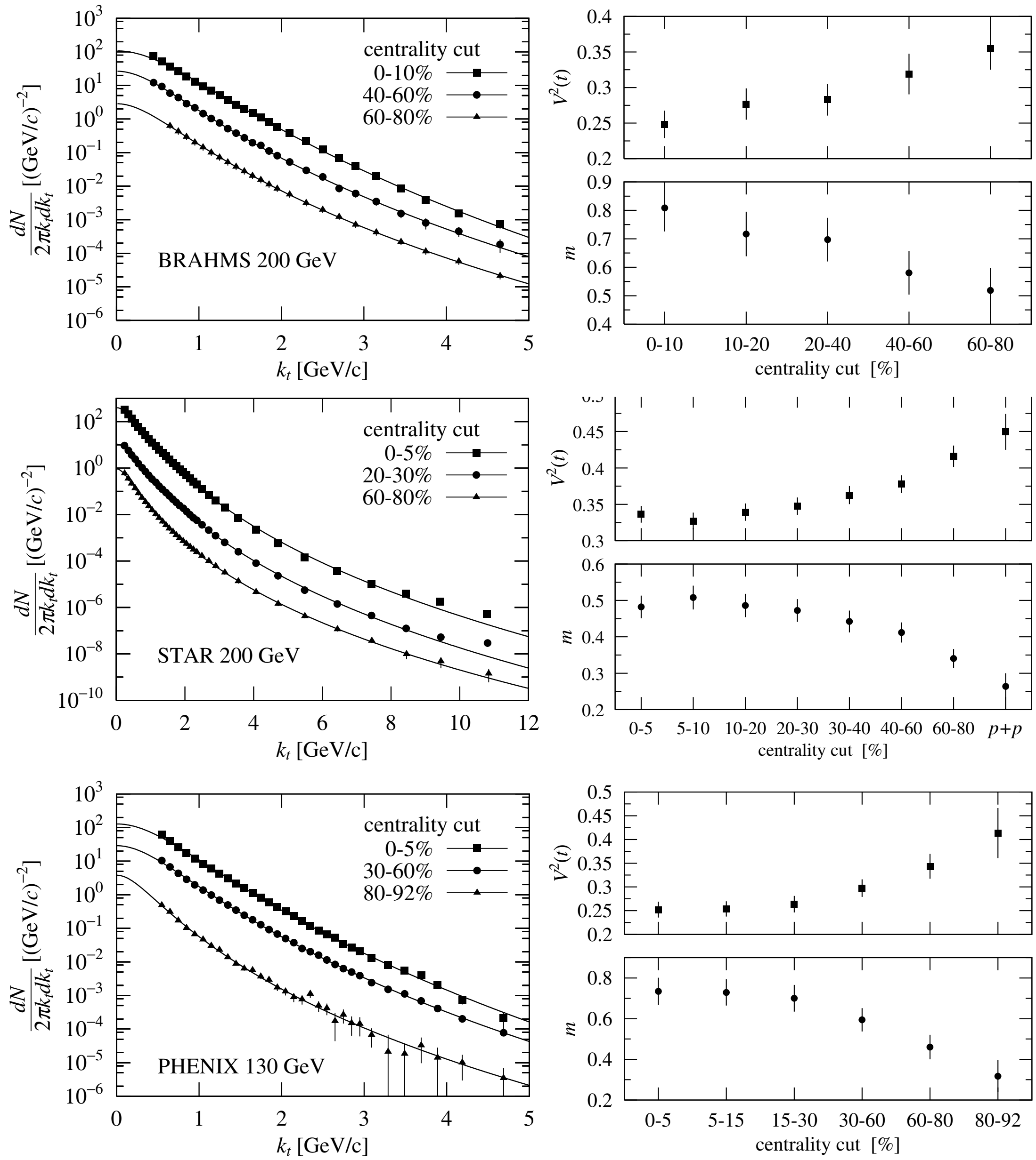

Fig. 4. Results of the application of the stochastic approach as given by (9) to the data for k_t distributions at $\sqrt{s_{NN}} = 200\,\mathrm{GeV}$ measured for different centralities by the BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130\,\mathrm{GeV}$ obtained by the PHENIX Collaboration [3]. Notice that the mass m is treated here as a free parameter, in a way similar as in [14]

fluctuation–dissipation relation, which in our case means that the measure of the size of diffusion (dissipation), $V_\mathrm{t}^2(t)$, can be expressed by the temperature T_0 and mass m:

$$V^2(t) = \frac{T_0}{m}. \qquad (11)$$

Therefore our results for V^2 shown in Fig. 4 (see the inlets), where $V(t)^2$ increases with increasing centrality, would indicate that the temperature T_0, obtained by applying Einstein's relation with m kept constant, would increases with centrality as well, contrary to what has been obtained above by applying the q-statistical approach. We have allowed then (following [14]) the mass m to be a free parameter and the best fit is obtained when m decreases with centrality; see the inlets in Fig. 4. The resulting temperature, $T_0 \simeq m \cdot V_\mathrm{t}^2$, behaves then in essentially the same way as a function of the centrality as in the q-statistical approach, cf., Table 3 and Fig. 5[5].

[5] It should be stressed here that for a constant value of the mass, $m = 0.14\,\mathrm{GeV}$ as used for the q-statistics case above, we would have obtained somewhat higher values of the χ^2's. In addition, it is interesting to observe at this point that the fact that we can fit he data within the modified stochastic approach only by allowing for a kind of "quasiparticles" of mass m, different for different centralities, corresponds in a sense to introducing the parameter q into the usual statistical

Table 3. Values of the characteristic parameters used to fit the data on k_t distributions at different centralities by using the stochastic approach as given by (9) and presented in Fig. 4 for the data at $\sqrt{s_{NN}} = 200$ GeV obtained by the BRAHMS [1] and STAR [2] Collaborations and at $\sqrt{s_{NN}} = 130$ GeV obtained by the PHENIX Collaboration [3]. The order of magnitude of the corresponding errors for T_0, δT_0, are listed below as well

C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
BRAHMS Coll.; $\delta T_0 = 0.008$–0.012; $\delta m = 0.024$–0.031				
0–10	39.9/23	140 ± 9	0.201	0.784
10–20	24.2/23	108 ± 7	0.199	0.725
20–40	20.1/23	72 ± 5	0.196	0.671
40–60	11.9/23	38 ± 4	0.185	0.577
60–80	4.06/22	4.3 ± 0.5	0.184	0.515
STAR Coll.; $\delta T_0 = 0.004$–0.006; $\delta m = 0.009$–0.014				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
0–5	221/32	484 ± 22	0.169	0.533
5–10	124/32	370 ± 18	0.170	0.547
10–20	121/32	310 ± 14	0.168	0.513
20–30	92.9/32	217 ± 10	0.168	0.498
30–40	89.9/32	158 ± 8	0.165	0.473
40–60	43.9/32	99.6 ± 5.0	0.157	0.419
60–80	22.3/32	43.2 ± 2.3	0.143	0.349
$p+p$	17.4/29	5.29 ± 0.63	0.126	0.298
PHENIX Coll.; $\delta T_0 = 0.020$–0.037; $\delta m = 0.058$–0.078				
C.C. (%)	χ^2/n.d.f.	C	T_0 (GeV)	m (GeV)
0–5	8.06/29	161 ± 34	0.185	0.734
5–15	5.61/29	124 ± 26	0.185	0.729
15–30	7.27/29	80 ± 18	0.185	0.700
30–60	3.49/29	39 ± 9	0.177	0.594
60–80	11.1/29	12 ± 4	0.158	0.460
80–92	9.01/29	6.1 ± 3.6	0.131	0.317

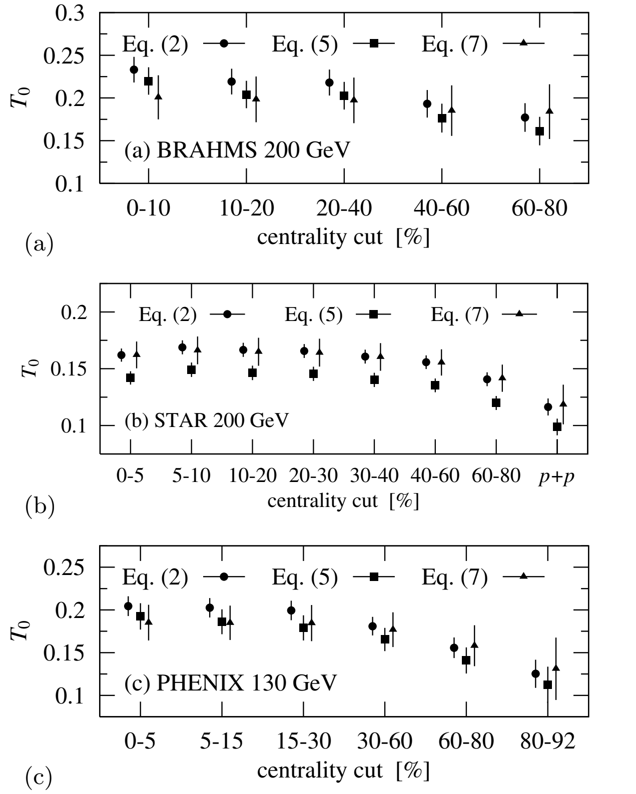

Fig. 5. Comparison of the temperatures of hadronization obtained by using different approaches as given by **a** (3) with $Q = q/(1-q)$; **b** (3) and $Q = 1/(1-q)$; **c** (9). In the latter case T_0 has been obtained from the values of V_t^2 and m was obtained in Fig. 4 by using Einstein's relation: $T_0 = m \cdot V^2(t)$

4 Concluding remarks

We have provided here a systematic analysis of recent RHIC data on k_t distributions [1–3] by using three different kinds of statistical approaches: the Hagedorn model [12], two versions of the modified statistical based on Tsallis statistics [5] and a suitable adaptation of the stochastic model proposed in [9]. We have found that the Hagedorn-type model widely used (for quick estimations) simplified version with $\rho(m) = 1$, which is then just a simple Boltzmann gas model with only one parameter, the temperature T_0, fails completely even for smaller k_t; cf. Table 4). However, these data can still be reasonably well fitted either by non-extensive extensions of the statistical model [5] or by the picture of some suitable diffusion process taking place in transverse rapidity space [9,10]. This is specially true if one limits itself to the $k_t < 5$ GeV/c range as the case of the BRAHMS [1] and PHENIX [3] data, the $k_t < 12$ GeV/c range considered in the STAR experiment [2] seems to be already too big to be fitted properly even with these two approaches (the corresponding values of the χ^2 are considerably bigger in this case and the values of the parameters obtained for the STAR and BRAHMS data, which were taken at the same collision and at the same energy, are also different).

As is shown in Fig. 5, the temperatures T_0 obtained in the modified statistical and stochastic approaches (with varying mass m) are essentially very similar to each other and follow the same dependence on the centrality, namely T_0 decreases when the collision is more peripheral. However, because the stochastic approach seems to be more dynamical than the q-statistical one (where the true dynamical origin of the non-extensivity parameter is not yet firmly established, see [6,15]), we regard as the most valuable our finding that stochastic approach [9,10,14] works so well and can serve to provide first simple estimations of any new data in the future. On the other hand we have also demonstrated that the two possible approaches using q-statistics are equivalent, at least in the frame of the limited phenomenological approach presented here. One should also stress at this point that the q-statistical approach offers unique information on the fluctuations in the system, which can be translated into information on its volume. Our results for AA and pp collisions taken to-

model. The possible dynamical origin and meaning of such variable mass is, however, still lacking.

Table 4. Comparison of investigated models: simple statistical model (i.e., Hagedorn model as given by (1) but with $\rho(m) = 1$, in which case it is just a simple statistical Boltzmann gas model with only one parameter, namely the temperature T_0), non-extensive Tsallis distribution (NETD) and Ornstein–Uhlenbeck process (O-U), using data on k_t distributions at $\sqrt{s_{NN}} = 200$ GeV obtained by the BRAHMS Collaboration [1] for smallest and largest centralities

		Simple statistical model, (1) with $\rho(m) = 1$			NETD (3) (with $Q = q/(1-q)$)			O-U (9)		
C.C. (%)		T_0 (GeV)	q	m (GeV)	T_0 (GeV)	q	m (GeV)	T_0 (GeV)	q	m (GeV)
0–10	χ^2/n.d.f.		177/23			10.2/23			39.9/23	
		0.302	–	–	0.232	1.043	–	0.201	–	0.784
60–80	χ^2/n.d.f.		567/22			2.76/22			4.06/22	
		0.325	–	–	0.175	1.084	–	0.184	–	0.515

gether with old results for e^+e^- annihilations indicate in this respect the distinct growth of the expected volume of interactions from the most elementary annihilation processes to the nuclear collisions.

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References

1. I. Arsene et al. [BRAHMS Collaboration], Phys. Rev. Lett. **91**, 072305 (2003)
2. J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. **91**, 172302 (2003)
3. K. Adcox et al. [PHENIX Collaboration], Phys. Lett. B **561**, 82 (2003)
4. Cf., for example, W. Broniowski, W. Florkowski, Phys. Rev. Lett. **87**, 272302 (2001), Acta Phys. Polon. B **35**, 779 (2004) and references therein
5. C. Tsallis, J. Stat. Phys. **52**, 479 (1988); in: Non-extensive Statistical Mechanics and its Applications, edited by S. Abe and Y. Okamoto, Lecture Notes in Physics LPN560 (Springer 2000). For the most recent developments in this field see http://tsallis.cat.cbpf.br/biblio.htm
6. W.M. Alberico, A. Lavagno, P. Quarati, Eur. Phys. J. C **12**, 499 (2000); O.V. Utyuzh, G. Wilk, Z. Włodarczyk, J. Phys. G **26**, L39 (2000); G. Wilk, Z. Włodarczyk, Physica A **305**, 227 (2002); F.S. Navarra, O.V. Utyuzh, G. Wilk, Z. Włodarczyk, Nuovo Cim. C **24**, 725 (2001); Phys. Rev. D **67**, 114002 (2003); Physica A **340**, 467 (2004); **344**, 568 (2004); Nukleonika **49**, (Supplement 2) (2004) S19
7. I. Bediaga, E.M. Curado, J.M. de Miranda, Physica A **286**, 156 (2000)
8. T. Wibig, I. Kurp, Int. J. High Energy Phys. **0312**, 039 (2003)
9. M. Biyajima, M. Ide, T. Mizoguchi, N. Suzuki, Prog. Theor. Phys. **108**, 559 (2002) [Addendum **109**, 151 (2003)]; M. Biyajima, T. Mizoguchi, Prog. Theor. Phys. **109**, 483 (2003); see also G. Wolschin, Eur. Phys. J. A **5**, 85 (1999)
10. M. Biyajima, M. Ide, M. Kaneyama, T. Mizoguchi, N. Suzuki, Prog. Theor. Phys. Suppl. **154**, 344 (2004); see also G. Wolschin, Phys. Lett. B **569**, 67 (2003); Phys. Rev. C **69**, 024906 (2004)
11. Minh Duong-van, P. Carruthers, Phys. Rev. Lett. **31**, 133 (1973)
12. R. Hagedorn, Nuovo Cim. Suppl. **3**, 147 (1965); Nuovo Cim. A **52**, 64 (1967); Riv. Nuovo Cim. **6**, 1983 (1983)
13. R. Hagedorn, J. Ranft, Suppl. Nuovo Cim. **6**, 169 (1968)
14. N. Suzuki, M. Biyajima, Acta Phys. Polon. B **35**, 283 (2004)
15. G. Wilk, Z. Wlodarczyk, Phys. Rev. Lett. **84**, 2770 (2000)
16. C. Beck, Physica A **286**, 164 (2000)
17. G. Wilk, Z. Wlodarczyk, Chaos Solitons Fractals **13**, 3, 581 (2001)
18. C. Beck, E.G.D. Cohen, Physica A **322**, 267 (2003)
19. C. Tsallis, R.S. Mendes, A.R. Plastino, Physica A **261**, 534 (1998)
20. A. Lavagno, Physica A **305**, 238 (2002); M. Rybczyński, Z. Włodarczyk, G. Wilk, Nucl. Phys. (Proc. Suppl.) B **122**, 325 (2003)